Review

→ 2D TI = QSH state

→ In 3DTI, SOC plays an important part.

$$H_{eff} = v\,\mathrm{sgn}(B)\,[\underbrace{p_y\sigma_y + p_z\sigma_z}_{\text{"real spin"}}] \qquad H_{graphene} = \vec{p}\cdot\underbrace{\vec{\sigma}}_{\text{"pseudospin"}}$$

Properties of Topological Insulators

① Surface state has a "spin-momentum" locking.

↳ e^-'s can go longer dist w/o scattering

any random defect, that does not break TRS → then any crystalline defect cannot "cause backscattering"

② TRS & Absence of Backscattering

$$|-k,\downarrow\rangle = \Theta\,|k,\uparrow\rangle \quad \text{"anti-unitary"}$$

for spin $\frac{1}{2}$, $\Theta^{\dagger} = -\Theta$ (on spin space) & $\Theta^2 = -1 \Rightarrow \Theta^{-1} = -\Theta$

now,

$$\langle -k,\downarrow|\,U\,|k,\uparrow\rangle = \left(\langle -k,\downarrow|\Theta^{\dagger}\right) U \left(\Theta|k,\uparrow\rangle\right)$$

$$= -\left(\langle -k,\downarrow|\Theta\right) U \left(\Theta|k,\uparrow\rangle\right)$$

$\langle\varphi|\left(\Theta^{\dagger}|\psi\rangle\right)$

$\left(= \langle\psi|\left(\Theta|\varphi\rangle\right)\right.$ $=$

$-\langle\varphi|\left(\Theta|\psi\rangle\right) = \langle\psi|\left(\Theta|\varphi\rangle\right)$

$\Theta U = U\Theta$

| if U has spin ½

comp.

$$\Theta\, U\, \Theta = -U$$

but if U lacks any spin 1/2 component,

$$\Theta U = U \Theta \xrightarrow{\Theta^2 = 1} \Theta U \Theta = U$$

$\therefore$ Assuming U lacks spin components,

$$\Theta U \Theta = U$$

$$\therefore \quad \langle -k,\downarrow | U | k,\uparrow\rangle = \left(\langle -k,\downarrow|\Theta^{+}\right) U \left(\Theta|k,\uparrow\rangle\right)$$

$$= -\langle -k,\downarrow| \left(\Theta U \Theta\right) |k,\uparrow\rangle$$

$$= -\langle -k,\downarrow| U |k,\uparrow\rangle$$

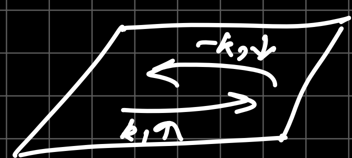
$$= 0$$

which ket does $\Theta U \Theta$ act on ?

$\therefore$ no backscattering is possible for a "nonmagnetic" impurities.

$\therefore$ unless U breaks TRS & is magnetic, there is no back-scattering.

$\hookrightarrow$ for e.g. = atomic defects, phonons etc.



& backscattering ≡ resistance $\hookrightarrow$ can power next gen. of electronic devices.

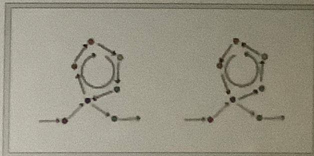
③ Weak Antilocalization

weak localization:- it happens when $L_{device} < L_\phi$ = phase coherence length.

$L_\phi \sim 100nm - 1\mu m$

Q. interference becomes an imp. effect & corrects diffusion transport (i.e. $L > l_\tau = v_F \tau$ mean free path)

General principle [edit]



Weak localization is due primarily to self-intersecting scattering paths

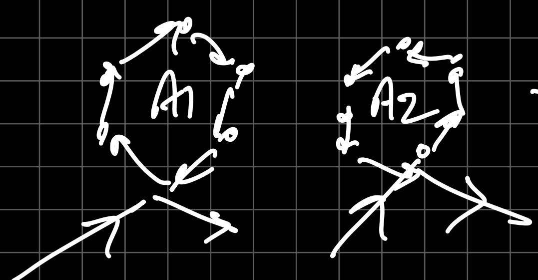
The effect is quantum-mechanical in nature and has the following origin: In a disordered electronic system, the electron motion is diffusive rather than ballistic. That is, an electron does not move along a straight line, but experiences a series of random scatterings off impurities which results in a random walk.

The resistivity of the system is related to the probability of an electron to propagate between two given points in space. Classical physics assumes that the total probability is just the sum of the probabilities of the paths connecting the two points. However quantum mechanics tells us that to find the total probability we have to sum up the quantum-mechanical amplitudes of the paths rather than the probabilities themselves. Therefore, the correct (quantum-mechanical) formula for the probability for an electron to move from a point A to a point B includes the classical part (individual probabilities of diffusive paths) and a number of interference terms (products of the amplitudes corresponding to different paths). These interference terms effectively make it more likely that a carrier will "wander around in a circle" than it would otherwise, which leads to an *increase* in the net resistivity. The usual formula for the conductivity of a metal (the so-called Drude formula) corresponds to the former classical terms, while the weak localization correction corresponds to the latter quantum interference terms averaged over disorder realizations.

The weak localization correction can be shown to come mostly from quantum interference between self-crossing paths in which an electron can propagate in the clock-wise and counter-clockwise direction around a loop. Due to the identical length of the two paths along a loop, the quantum phases cancel each other exactly and these (otherwise random in sign) quantum interference terms survive disorder averaging. Since it is much more likely to find a self-crossing trajectory in low dimensions, the weak localization effect manifests itself much stronger in low-dimensional systems (films and wires).[2]

Weak anti-localization [edit]

In a system with spin-orbit coupling the spin of a carrier is coupled to its momentum. The spin of the carrier rotates as it goes around a self-intersecting path, and the direction of this rotation is opposite for the two directions about the loop. Because of this, the two paths along any loop interfere *destructively* which leads to a *lower* net resistivity.[3]

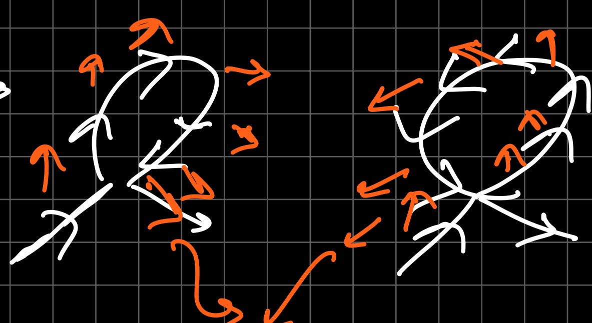


→ $A_1 / A_2 \sim$ const

∴ e^{is} & e^{-is} type intuition

survive disorder averaging

Antilocalizatⁿ →



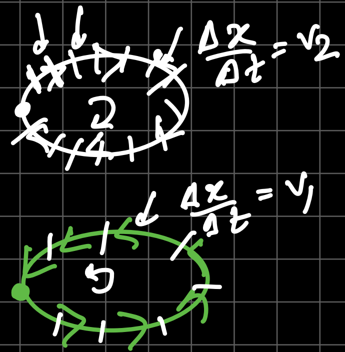
[destructive (∵ opp) interference occurs ← diff, not necessarily cancelling each other

Let A_1 be the amplitude for $\circlearrowright$ & A_2 for $\circlearrowleft$

Then $P(\text{for a cyclic detour}) = |A_1 + A_2|^2$

now from PI, $A_1 \sim e^{iS_1}$ where $S_1 = \oint \frac{m\dot{x}^2}{2} dt$

similarly $A_2 \sim e^{iS_2}$ where $S_2 = \oint \frac{m\dot{x}^2}{2} dt$



$v_1 = -v_2$

but $v_1^2 \Delta t = v_2^2 \Delta t$

$\rightarrow \underline{\underline{S_1 = S_2}}$

$\therefore A_1 \sim e^{i\phi}$ & $A_2 \sim e^{i\phi}$ } same

$\therefore |A_1 + A_2|^2 = 4A^2$ (A_1 & A_2 have same phase)

$\therefore$ large likelihood of just a "wandering" <u>around path.</u>

Break constructive int $\Rightarrow$ apply $\vec{B} \neq 0$

$\hookrightarrow$ aharonov Bohm phase of $\oint \vec{A} \cdot d\vec{l}$ is opposite for both

interference is now given by

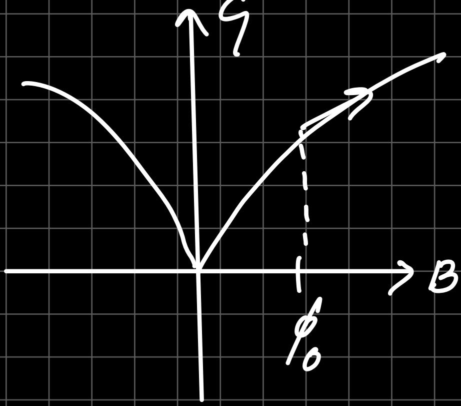
$P(\text{loop detour}) = \underbrace{2A^2 + 2A^2}_{\downarrow} \cos(BS)$

$S \hookrightarrow$ area of loop

$\hookrightarrow$ in units of ϕ_0

$P_{detour} \downarrow, \ \sigma \uparrow$

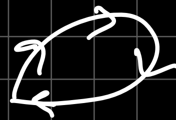
<u>"antilocalizatn"</u>



→ provided the path is smaller than L_ϕ

→ typical metallic behaviour

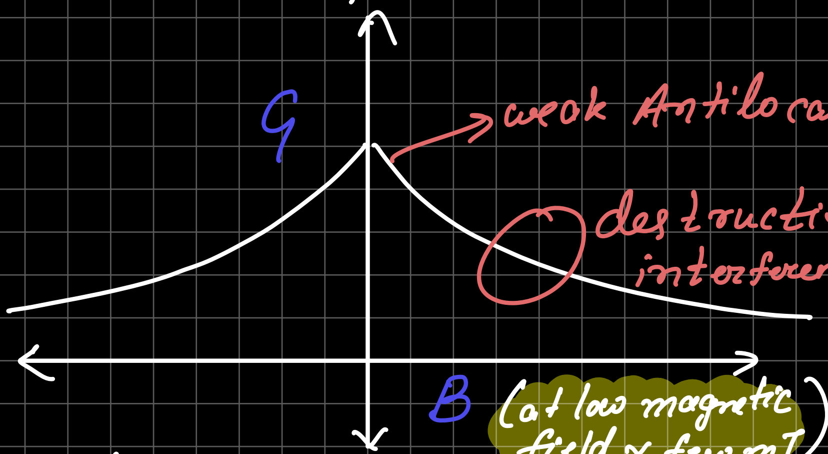
(why weak? → extreme disorder, Anderson localizat^n)



For TIs, weak-Antilocalization

$H_{eff} \sim (\vec{p} \times \vec{\sigma})_z$ → no B needed, to be introduced for opp.

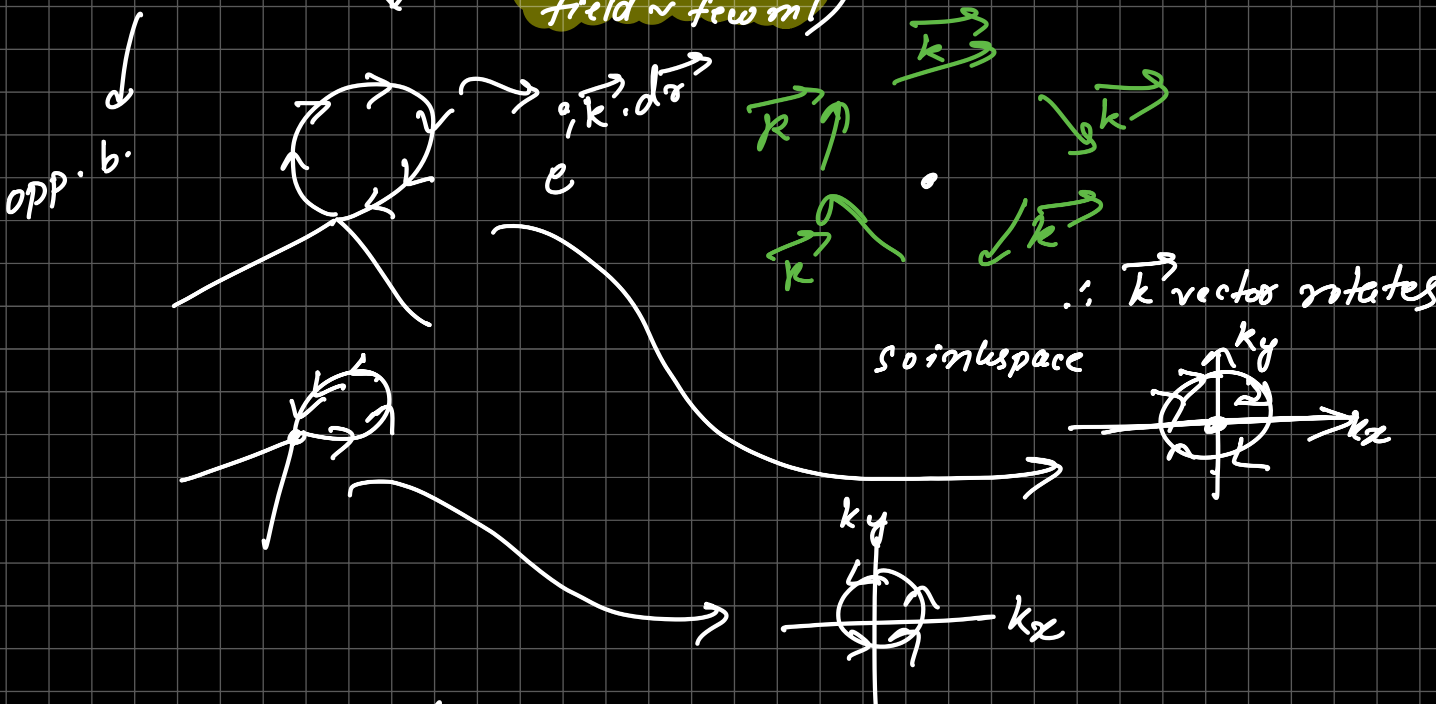
↳ BP $= \pi$



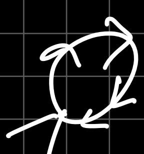
(not necessarily due to Berry phase)

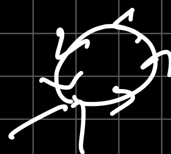
↓

SOC in some other high Z materials

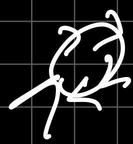
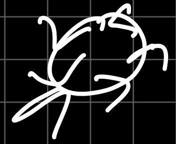


now if there did not exist B.P.

 $\sim e^{i\oint \vec{k}\cdot d\vec{r}}$

 $\sim e^{i\oint(-\vec{k})\cdot(-d\vec{r})}$

however B phase

 $\sim e^{i\gamma}$  $\sim e^{-i\gamma}$

so it can lead to destructive int. (but how, if it accumulates π & $-\pi$ phase?)

ψ accumu π for a CW rotat^n.
" " $-\pi$ " " ACW rota^n $\Big] \Delta\phi = 2\pi$
so why destructive

why ???

How does one measure Berry phase?

SdH oscillations

4–5 expts

QAHE

QAHE